Dynamic Consumer Cash Inventory Model

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Introduction

- rapid technological developments in the payment industry
  - $\rightarrow$ new payment instruments become available/are increasingly adopted
  - $\rightarrow$ consumers balance use of multiple instruments (cash, cards, mobile payments)
- cash usage at the point of sale (POS) has been declining over the past decade
  - $\rightarrow$ calls to abandon cash (Rogoff 2017), but retains market share (Henry et al. 2024)
  - $\rightarrow$ cash puzzle: demand for bank notes is steadily increasing (Engert et al. 2019)
    - $\Rightarrow$ continued role for central banks to ensure accessibility of cash for entire populace
- infrastructure to access cash constantly evolving
  - $\rightarrow$ # of bank branches (predominant cash access point) in Canada peaked around 2013
  - $\rightarrow$ ongoing branch closure programs (e.g., Desjardins, Laurentian), reduction of up to 30% of branches

$\Longrightarrow$ Who are the winners & losers from these changes?

Contribution & Findings

- structural model of dynamic cash inventory management
  - → accounts for payment choice at point of sale: cash vs. non-cash
  - → estimation exploits multiple waves (2009, 2013, 2017) of detailed survey & diary data
  - → address role of changing infrastructure
- explicitly account for consumer heterogeneity
  - → representative consumer vs. fully flexible estimation approach: heterogeneity matters
  - → substantial heterogeneity even within demographic groups (old/young/rich/poor/urban/rural/...)
- consumer heterogeneity translates to bimodal response to changes in infrastructure
  - → substantially reduce cash use vs. infrequent but larger withdrawals & holdings
  - → younger and less affluent households <u>forced</u> to substitute away from cash despite preferences

Literature (non-exhaustive)

Baumol-Tobin type inventory models

→ Baumol (1952), Tobin (1956), Smith (1986), Whitesell (1989), Lippi and Secchi (2009) Alvarez and Lippi (2009; 2017), Chen et al. (2021) ...

Technological Innovation & Payment Choice

→ Arango et al. (2015), Koulayev et al. (2016), Chen et al. (2017), Wakamori and Welte (2017), Huynh et al. (2020; 2022), Engert et al. (2024) ...

Nexus of Cash Inventory Management & Payment Choice

→ Alvarez and Lippi (2017), Briglevics and Schuh (2020), Moracci (2022)

Cash Puzzle & Role of Cash

→ Rogoff (2017), Greene and Schuh (2017), Henry et al. (2018), Engert et al. (2019)

Heterogeneity Methods

→ Ackerberg (2009), Nevo et al. (2016), Malone et al. (2021), McManus et al. (2022)

Model: Overview (Parameters of Interest)

- Preferences (α): consumers want to use mix of cash & non-cash for transactions
  - $\rightarrow$ parameter on cash part of log-linearized Cobb-Douglas utility for payment choice at POS
  - $\rightarrow$ cash requires cash inventories comprised of cash holdings from previous period & withdrawals
- Costly withdrawals (F): interacted with distance (d) to cash access points
  - $\rightarrow$ distance measure d: taken from data, proxy for bank branch density in consumer's FSA Details
  - $\rightarrow$ cost of withdrawals depends on infrastructure (d) *and* preference/perception (F)
- Holding costs (γ): security, foregone interest, ...

$\Longrightarrow$ trade-off between frequent withdrawals and larger holdings to facilitate cash use at POS

One-Slide Model

4-slide version

- consumers use cash (c_{it}) or non-cash $(s_{it} - c_{it})$ to settle exogenous noisy consumption s_{it}
  - $\rightarrow$ cash uses inventory h_{it-1} carried over or costly withdrawal $w_{it} > 0$

$$\max_{(c_{it}, w_{it})\forall t} \overbrace{\mathbb{E} \sum_{t=0}^{\infty} \beta^t U\left(h_{it-1}, s_{it}, d_i, w_{it}, c_{it}\right)}^{\text{discounted sum of flow utilities}} \Longleftarrow \quad \text{reflects } \alpha, F, \gamma \text{ (log-linearized Cobb-Douglas)}$$

$$s.t. \underbrace{0 \leq c_{it} \leq h_{it-1} + w_{it}}_{0 \leq \text{ cash use } \leq \text{ cash } + \text{ withdrawal}}, \quad \underbrace{c_{it} \leq s_{it}}_{\text{cash use } \leq \text{ expenditure}}, \quad \underbrace{w_{it} \geq 0}_{\text{withdrawals nonnegative}}, \underbrace{h_{i,-1} = 0}_{\text{initial cash}}$$

- key: (c_{it}, w_{it}) impact future only via (implicit) choice of cash inventory h_{it}
  - $\rightarrow$ recast problem as choice of h_{it} implying conditionally optimal (c^*_{it}, w^*_{it})
  - $\rightarrow$ analytical characterization of conditional in-period choice
- approximate value function by backward induction using 6-months planning horizon
  - $\rightarrow$ $V(h_{it-1}, s_i, d_i, \epsilon_{it}) = \max_{h_{it} \geq \max\{0, h_{it-1} - s_{it}\}} \left\{\tilde{u}\left(h_{it}; h_{it-1}, s_{it}, d_i\right) + \beta \int V\left(h_{it}, s_i, d_i, \epsilon_{it+1}\right) dF_\epsilon\right\}$
  - $\rightarrow$ yields policy $h^*_{it}(h_{it-1}, s_i, d_i, \epsilon_{it})$ pinning down (w^*_{it}, c^*_{it})

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Data: Cash Withdrawal and Usage

- combine data from 2009, 2013 and 2017 Method of Payments Surveys
  - $\rightarrow$ survey questionnaire: demographics (age, income, location, revolver status)
  - $\rightarrow$ three-day diary: all transactions made at the POS $\rightarrow$ aggregate by day

Variable	Mean	Median	Min	Max	s.d.
Withdrawal level	141.29	100.00	0.00	1000.00	134.72
Withdrawal frequency/day	0.10	0.07	0.03	1.00	0.08
Cash holdings	63.35	40.00	0.00	500.00	70.00
Cash holdings at withdrawal	17.90	10.00	0.00	200.00	22.92
Daily total spending	84.04	50.47	0.00	977.00	105.01
Daily cash use	14.74	2.00	0.00	760.00	35.99
Cash/Total spending	0.31	0.04	0.00	1.00	0.41

Notes: Based on 3424 individual consumers across three waves (2009, 2013, 2017).

- exogenous expenditures: categorize consumers according to 30 spending types $(E[s_i], \sigma^2_{s_i})$
  - $\rightarrow$ spending type reflects total expense, *not* methods of payment used
  - $\rightarrow$ augment with consumer-specific distance measure (inverse density of branches) Details

Data: Estimation procedure I

Factual Data by Year

Variable of Interest	Overall	2009	2013	2017
Average daily cash use	14.72	15.88	15.56	12.49
Average cash withdrawal	147.80	146.25	143.54	154.99
Probability of withdrawal	0.10	0.11	0.11	0.08
Average cash holding	69.75	66.45	70.24	72.17

- model predictions given parameter vector (α, F, γ)
  - → obtained by averaging over 100×183 day simulations
  - → compare prediction vs. data to obtain error terms

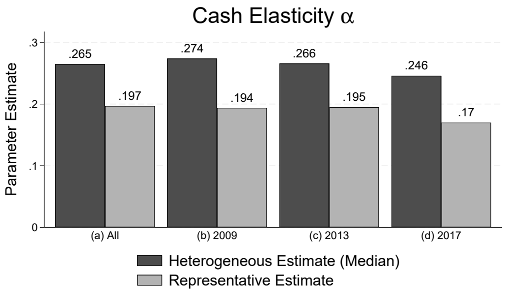
(i)	withdrawal amount given withdrawal:	$\varepsilon_{1,it} = \mathbb{E}\left[w_{it} \vert w_{it} > 0\right] - \mathbb{E}\left[\widehat{w_{it} \vert w_{it}} > 0\right]$
(ii)	withdrawal frequency:	$\varepsilon_{2,it} = \mathbb{E}\left[\Pr\left(w_{it} > 0\right)\right] - \mathbb{E}\left[\widehat{\Pr\left(w_{it}\right.} > 0)\right]$
(iii)	level of cash holdings:	$\varepsilon_{3,it} = \mathbb{E}\left[h_{it}\right] - \widehat{\mathbb{E}\left[h_{it}\right]}$
(iv)	level of cash use:	$\varepsilon_{4,it} = \mathbb{E}\left[c_{it}\right] - \widehat{\mathbb{E}\left[c_{it}\right]}$

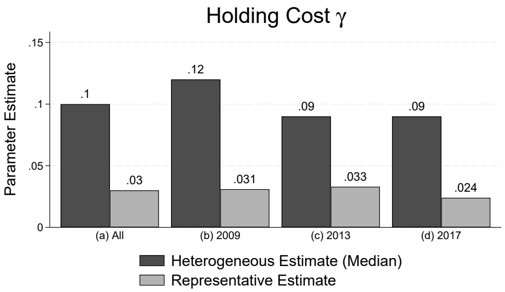
  - → 12 moments via interaction with 3 instruments
    - ∗ constant term, distance measure, spending level

Data: Estimation procedure II

- estimation contrasts representative approach with fully heterogeneous approach
  - → representative: sample/demographic group/year-specific $(\alpha_g, F_g, \gamma_g)$
    - ∗ estimation via GMM using stacked moment conditions
    - ∗ implies optimal 2nd-stage weighting matrix
    - ! consumer spending heterogeneous (30 types), only preferences homogenous (within-group)
  - → heterogeneous: consumer-specific $(\alpha_i, F_i, \gamma_i)$
    - ∗ minimize weighted sum of squared moments (in spirit of Ackerberg 2009, Malone et al. 2021)
    - ∗ candidates: 100'000 parameter tuples obtained via Halton Draws
    - ∗ use weighting matrix from representative approach to account for different moment scales
- report results for both approaches
  - → scale holding cost γ by 100 to facilitate visualization

Takeaway I: Accounting for heterogeneity matters!





- median estimates' trend qualitatively similar to representative approach for α, γ
  - $\rightarrow$ cash elasticity and inventory cost both decline over time
  - $\rightarrow$ notable level differences

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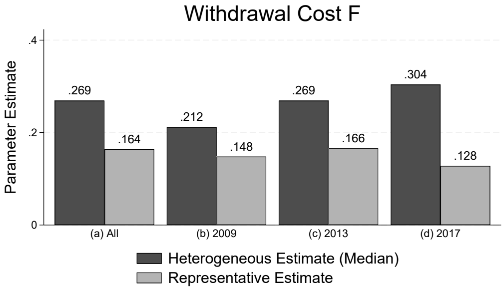
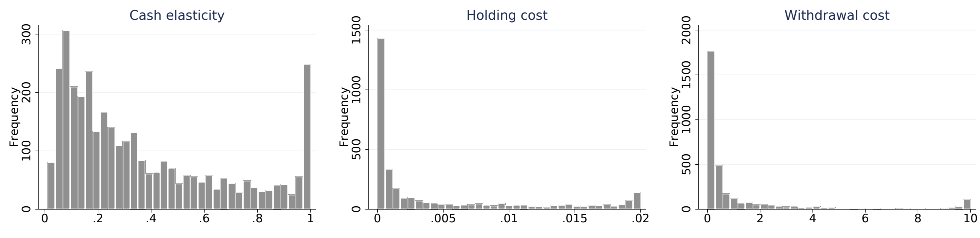


- qualitatively different trend for withdrawal cost parameter F
  - $\rightarrow$ upward trend for heterogeneous estimates
  - $\rightarrow$ inverse U-shaped trend for representative estimates

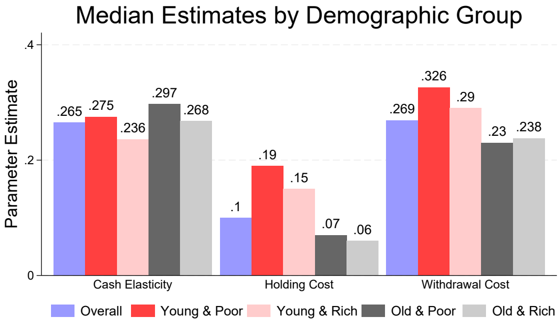
Takeaway I: Accounting for heterogeneity matters!

Distribution of α_i (left), γ_i (center), F_i (right) — All years



- heterogeneous approach reveals substantial heterogeneity
  - → cash preference α_i bi-modal but smooth between 0 and 1
  - → holding cost γ_i & withdrawal cost F_i exhibit very long tail
- model fit: heterogeneous approach outperforms representative approach
  - → true even after segmenting into finer groups (demographic group per year)
  - → representative approach unable to match increased cash holdings observed in the data

Takeaway II: Heterogeneity is partially explained by demographics!



- richer and older people use more cash but only age relation driven by preference
- older households have lower costs (both withdrawal and holding/inventory)
  - → cash elasticity increases over time for older & poorer consumers

Details

Takeaway III: Heterogeneity matters for response to Infrastructure Changes!

- evaluate factual infrastructure changes
  - → # of bank branches peaked in 2013, reflected in our sample
  - → How much cash would consumers have used in other years (relative to 2009)?

▸ Details

Infrastructure	All	Urban	Rural	Y&P	Y&R	O&P	O&R	Substantial*
2009	-	-	-	-	-	-	-	-
2013	0.99%	1.25%	-0.22%	1.51%	2.19%	0.65%	0.03%	-0.61%
2017	-0.12%	0.16%	-1.47%	-0.65%	1.80%	-0.22%	-1.08%	-15.21%

- moderate average impact of infrastructure changes on cash use on average
  - → some indications of urban/rural divide
  - → broadly follows infrastructure trend across demographic groups
    - ∗ younger, affluent consumers increase cash use
  - → 10-times larger impact following substantial changes ($\Delta d > 20\%$, 78 consumers)
    - ∗ goes hand in hand with increase in cash holdings

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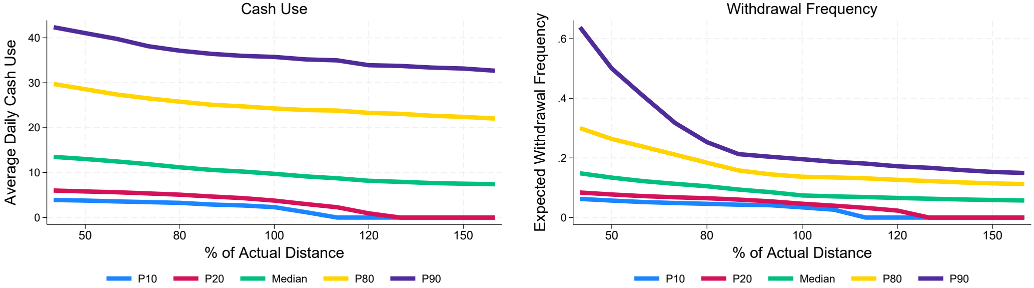
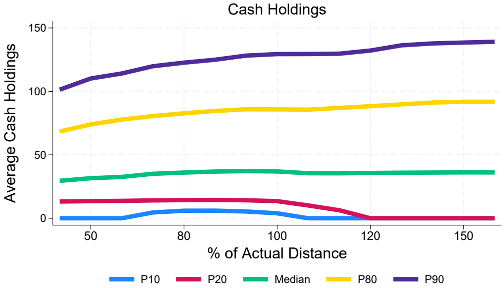


- Counterfactual: perturb distance to access-to-cash infrastructure for entire sample
  - → withdrawal costs ⇑ $\implies$ cash use ⇓ accompanied by less frequent withdrawals
  - → some consumers stop using cash as it gets too expensive to withdraw; $\approx 10\%$ (25%) increase sufficient for 10% (20%) of consumers to stop use of cash at POS

Takeaway III: Heterogeneity matters for response to Infrastructure Changes!



- cash holdings indicative of bimodality in responses
  - → continued cash users (P80+) hold more cash to economize on withdrawals
  - → cash avoiders (P20-) stop using cash as withdrawals become too costly

Takeaway III: Heterogeneity matters for response to Infrastructure Changes!

Estimates & demographics by response to 25% increase in distance to access-to-cash infrastructure

Variable	**Cash non-users**	**Cash users**		
		all	decreased holdings	increased holdings
	(1)	(2)	(3)	(4)
Cash elasticity α	0.368	0.374	0.328	0.532
Cash holding cost γ	0.011	0.003	0.003	0.003
Withdrawal cost parameter F	2.660	1.152	0.970	1.776
Age	43.259	49.667	49.006	51.927
Income	46179.856	48063.337	47501.312	49982.079
Revolver	0.249	0.197	0.208	0.159
Urban	0.878	0.841	0.823	0.901
Young & Poor	0.342	0.251	0.265	0.203
Young & Rich	0.283	0.202	0.203	0.195
Old & Poor	0.254	0.370	0.362	0.398
Old & Rich	0.121	0.177	0.169	0.204
Observations	696	2465	1907	558

- continued cash-users (cash holdings⇑, withdrawal amt.⇑) are older (& more affluent)
  - → extensive margin: no difference in cash elasticity relative to cash non-users, but lower costs
  - → intensive margin: preference & costs determine response
- younger and poorer consumers phase out cash use as cash becomes too costly for them

Robustness/Extensions/Other Results

- paper: several other analyses, bigger emphasis on distributional impact
  - → branch closures largely unrelated to consumer demographics & preferences
- robustness checks
  - → different planning horizons for consumers, # of points consumer value function is solved at
  - → alternative definitions of consumer spending types and distance measures
  - → structure of the weighting matrix, application to heterogeneous approach
- conceptually feasible extensions
  - → inventory model with deposits (not on equilibrium path)
  - → estimating discounting/risk aversion (in progress)
    - ∗ heavily overidentified model (3 parameters, 12 moments)
    - ∗ evidence in favor of consumer heterogeneity (Fulford and Schuh 2017)
  - → different impact of distance, free withdrawals, . . .

Conclusion

- structural dynamic cash inventory model
  - $\rightarrow$ trade-off between more frequent withdrawals and larger cash holdings
  - $\rightarrow$ choice between cash and non-cash
  - $\rightarrow$ account for infrastructure affecting costs of withdrawal
- estimate model using detailed data on consumer behavior
  - $\rightarrow$ accounting for individual consumer heterogeneity crucial
  - $\rightarrow$ bi-modality in consumer responses to increased costs of withdrawing cash
  - $\rightarrow$ younger & less affluent consumers bear brunt of impact: cash use $\rightarrow 0$ due to cost (not preference)

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Model: Setup I

Return

- discrete time, $t = 0, 1, \ldots, \infty$ (infinite horizon)
- Consumers $i = 1, \ldots, N_i$:
  - $\rightarrow$ cash inventories, $h_{it-1} \geq 0$, from previous period
  - $\rightarrow$ exogenous consumption demand $s_{it} = s_i + \varepsilon_{it}$, $\varepsilon_{it} \overset{iid}{\sim} N\left(0, \sigma^2_{si}\right)$
  - $\rightarrow$ allocate payments for consumption b/w cash, $c_{it} \geq 0$, and non-cash, $s_{it} - c_{it} \geq 0$
  - $\rightarrow$ can withdraw cash, $w_{it} \geq 0$, at fixed cost depending on F_i and d_i
    - $*$ d_i: distance to branch network (data), F: scaling parameter (estimated)

$$\max_{(c_{it}, w_{it})\forall t} \overbrace{\mathbb{E}\sum_{t=0}^{\infty} \beta^t U\left(h_{it-1}, s_{it}, d_i, w_{it}, c_{it}\right)}^{\text{discounted sum of flow utilities}}$$

$$s.t.\ \underbrace{0 \leq c_{it} \leq h_{it-1} + w_{it},}_{0\ \leq\ \text{cash use}\ \leq\ \text{cash}\ +\ \text{withdrawal}} \quad \underbrace{c_{it} \leq s_{it}}_{\text{cash use}\ \leq\ \text{expenditure}} \quad , \quad \underbrace{w_{it} \geq 0}_{\text{withdrawals nonnegative}} \quad , \underbrace{h_{i,-1} = 0}_{\text{initial cash}}$$

Model: Setup II

Return

- flow utility: log-transformed Cobb-Douglas

$$u\left(h_{it-1}, s_{it}, d_i, w_{it}, c_{it}\right) = \overbrace{\alpha \ln\left(1 + c_{it}\right) + \left(1 - \alpha\right) \ln\left(1 + s_{it} - c_{it}\right)}^{\text{flow utility}}$$
$$\underbrace{-F \times \mathbb{1}_{w_{it} > 0} \ln(1 + d_i)}_{\text{withdrawal cost}} \underbrace{-\gamma\left(h_{it-1} + w_{it} - c_{it}\right)}_{\text{holding cost}}$$

→ α parametrizes cash preference

→ d_i reflects distance to branch network (from data)

→ F parametrizes scale of withdrawal cost

→ γ parametrizes holding cost

Model: Setup II

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→ d_i reflects distance to branch network (from data)

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→ γ parametrizes holding cost

Model: Dynamics

◂ Return

- recursive formulation of the dynamic programming problem

$$V\left(h_{it-1}, s_i, d_i, \varepsilon_{it}\right) = \max_{c_{it}\geq 0, w_{it}\geq 0} \left\{ u(h_{it-1}, s_i + \varepsilon_{it}, d_i, w_{it}, c_{it}) + \beta \int V(h_{it}, s_i, d_i, \varepsilon_{it+1}) dF_\varepsilon \right\}$$

$$s.t.\ c_{it} \leq \min\{h_{it-1} + w_{it}, s_{it}\}, h_{it} = h_{it-1} + w_{it} - c_{it}$$

→ (c_{it}, w_{it}) impact future only via (implicit) choice of cash inventory h_{it}

* flow utility & withdrawal cost depend on (c_{it}, w_{it})
* holding cost & future value depend on h_{it}

→ recast problem as choice of h_{it} (Δh_{it}), inducing conditionally optimal (c_{it}, w_{it})

* analytical characterization of $(c^*_{it}(\Delta h_{it}), w^*_{it}(\Delta h_{it}))$
* withdrawal $w_{it} > 0$ triggered by (i) depleted cash reserves or (ii) large consumption shock
* mix of cash and non-cash throughout; withdrawal before cash is depleted

Dynamically Optimal Choice

Return

- given (w^*_{it}, c^*_{it}) we obtain $\tilde{u}(h_{it}; h_{it-1}, s_{it}) = u(h_{it-1}, s_{it}, d_i, c^*_{it}(h_{it}), w^*_{it}(h_{it}))$ and thus

$$V(h_{it-1}, s_i, d_i, \epsilon_{it}) = \max_{h_{it} \geq \max\{0, h_{it-1} - s_{it}\}} \left\{ \tilde{u}\left(h_{it}; h_{it-1}, s_{it}, d_i\right) + \beta \int V\left(h_{it}, s_i, d_i, \epsilon_{it+1}\right) dF_\epsilon \right\}$$

  → Bellman equation with current cash holdings as state & future cash holdings as control

- approximate $V(\cdot)$ by backward induction using a 183-period (6 months) planning horizon
  → robust to perturbations in length of planning horizon, discount factor ($\beta \approx 0.95$)
  → yields policy $h^*_{it}(h_{it-1}, s_i, d_i, \epsilon_{it})$ pinning down (w^*_{it}, c^*_{it})
  → implied behavior in line with theoretical predictions
    * withdrawal triggered by depleted cash reserves or large expenditure s_{it}
    * costly withdrawal implies fixed post-consumption cash inventory $\tilde{h} = \arg\max_h E[V(\cdot)] - \gamma \cdot h$

Dynamically Optimal Choice

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Model: In-period solution

Return

- ideal cash usage: $\tilde{c}(s) = \arg\max_{s \geq c \geq 0} \alpha \ln[1 + c] + (1 - \alpha) \ln[1 + s - c]$
- 3 regions depending on Δh_{it}
  - $\rightarrow$ $\Delta h_{it} < -\tilde{c}(s_{it})$: desired cash reduction exceeds ideal cash use
    - $*$ only use cash ($c_{it} = -\Delta h_{it}$), no withdrawal
    - $*$ withdrawal would be associated with even more excessive cash usage
  - $\rightarrow$ $\Delta h_{it} > 0$: increase cash holdings
    - $*$ withdrawal necessary, so $c_{it} = \tilde{c}(s_{it})$
  - $\rightarrow$ $\Delta h_{it} \in (-\tilde{c}(s_{it}), 0]$: intermediate reduction of cash holdings
    - $*$ trade-off b/w withdrawal & sub-optimal cash usage
    - $*$ either withdraw & implement ideal cash use...
    - $*$ or distort (reduce) cash use to avoid withdrawa
  - $\rightarrow$ withdrawal triggered by (i) depleted cash reserves or (ii) large consumption shock

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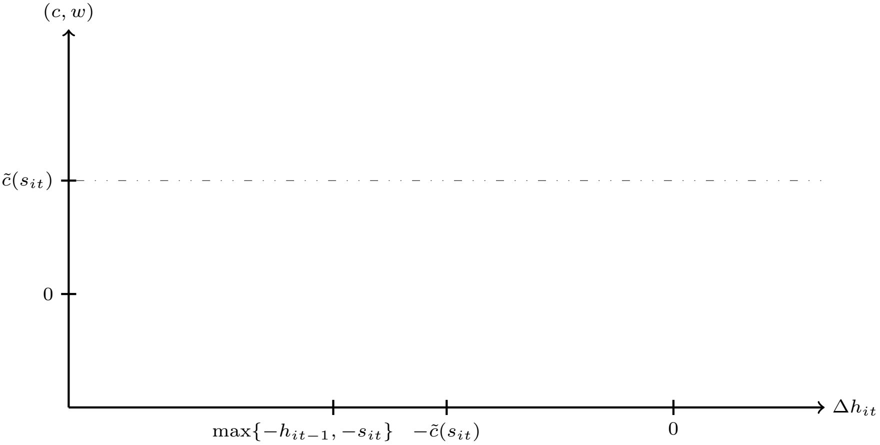
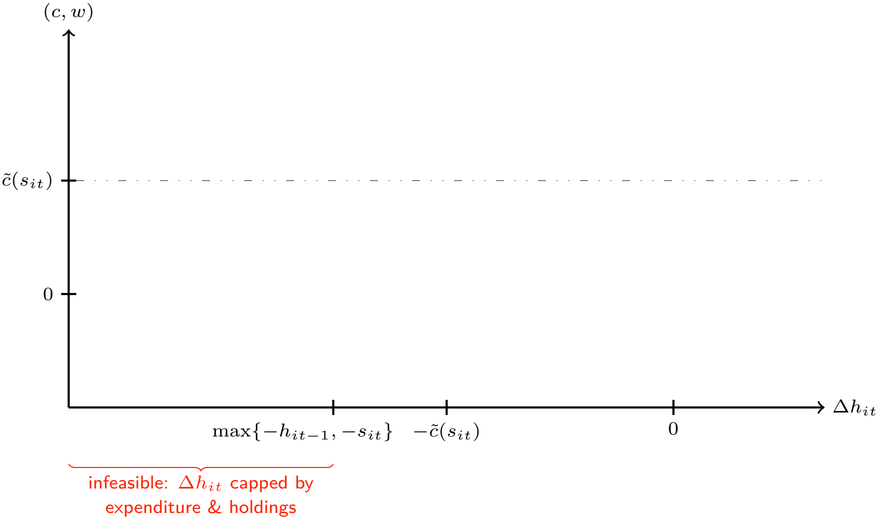


Model: In-period solution

Return



Model: In-period solution

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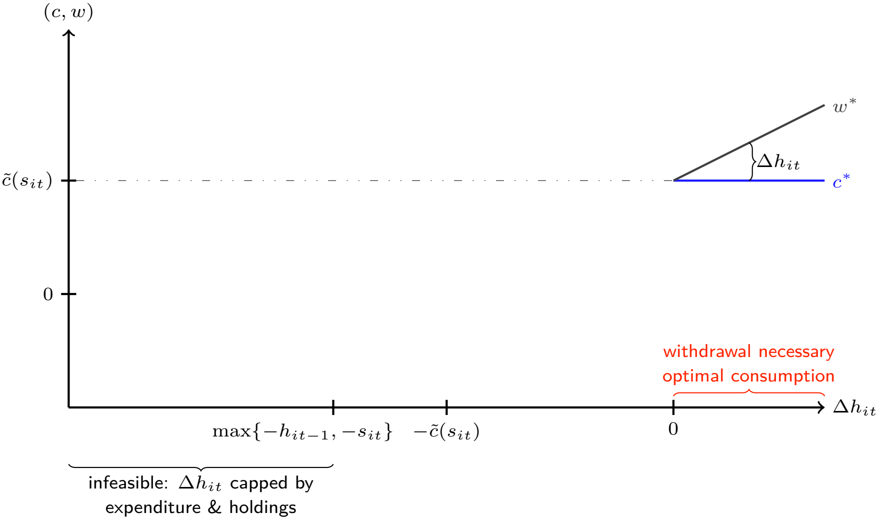

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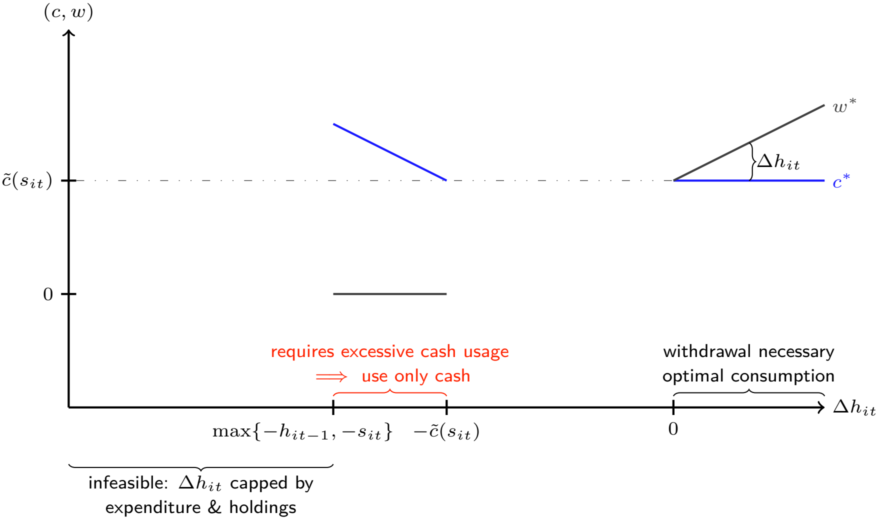

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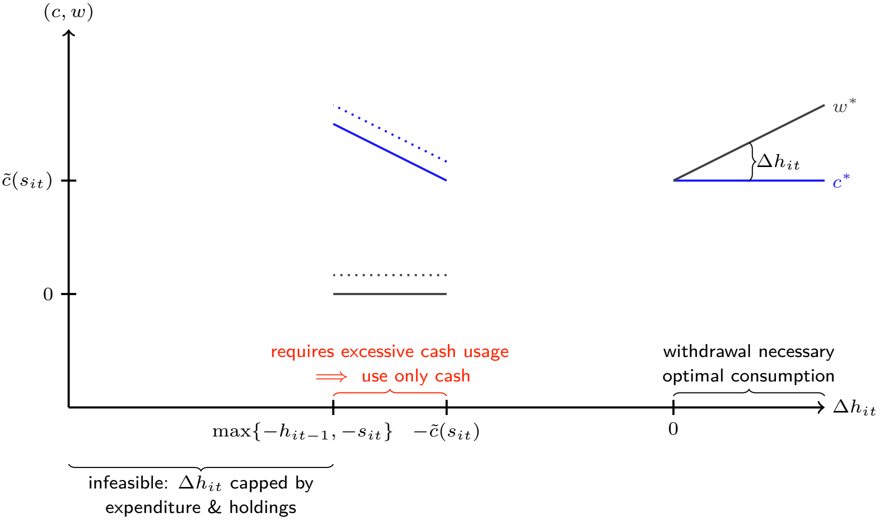

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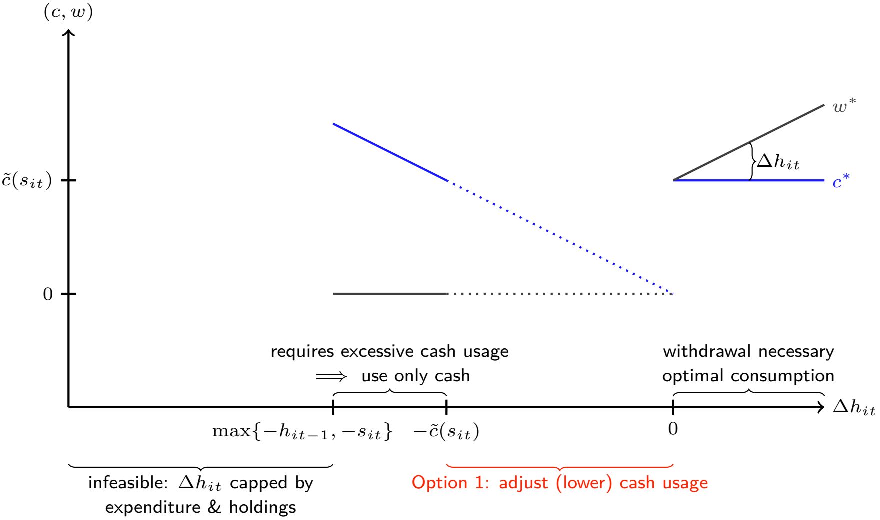

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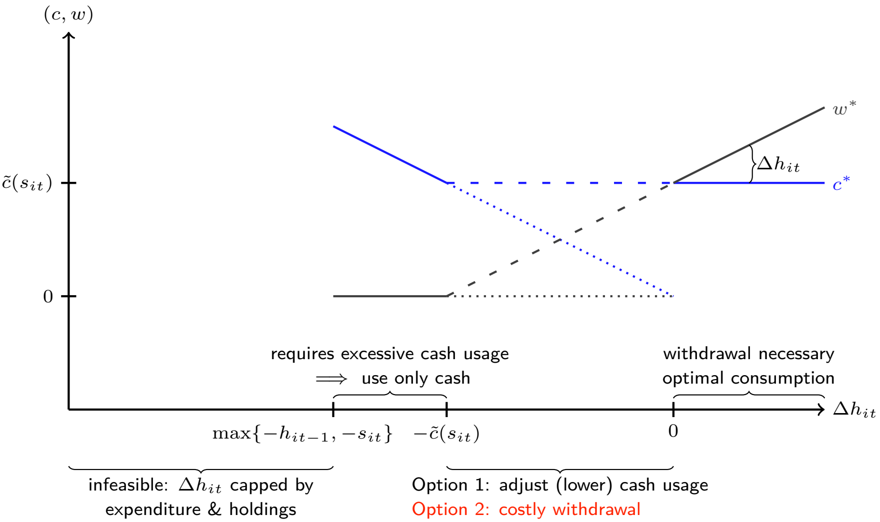

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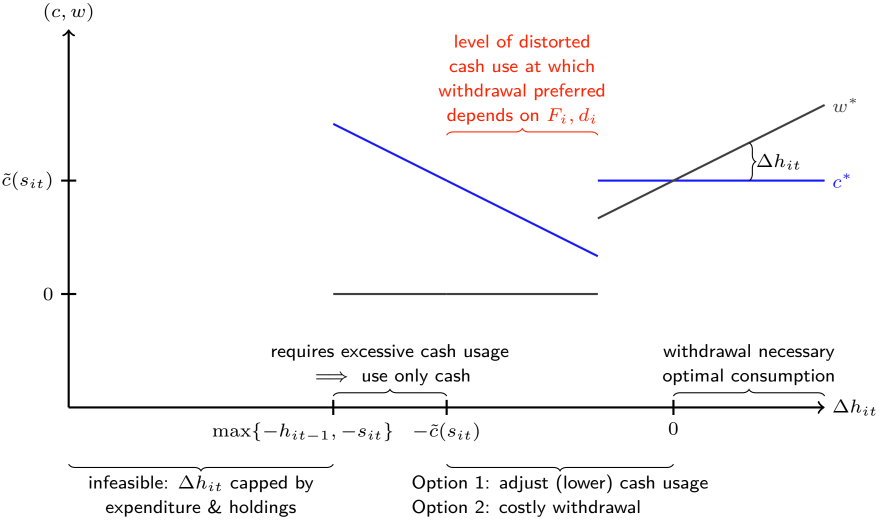

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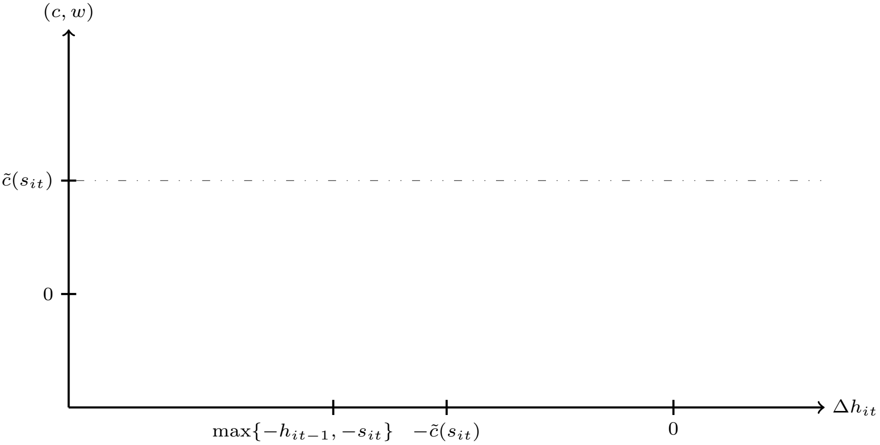
Return



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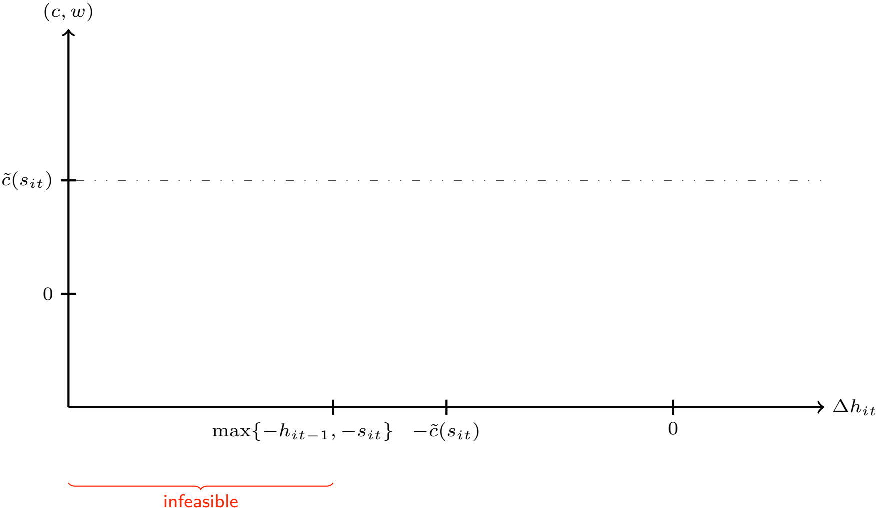

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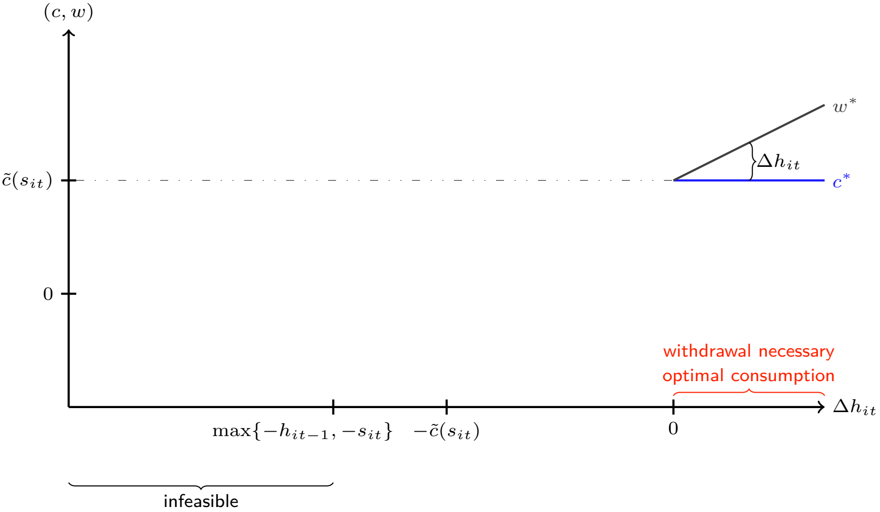

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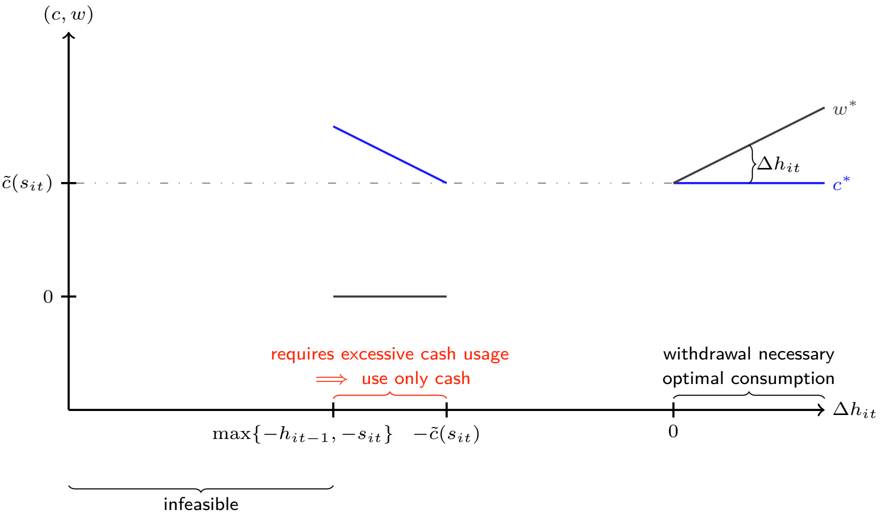
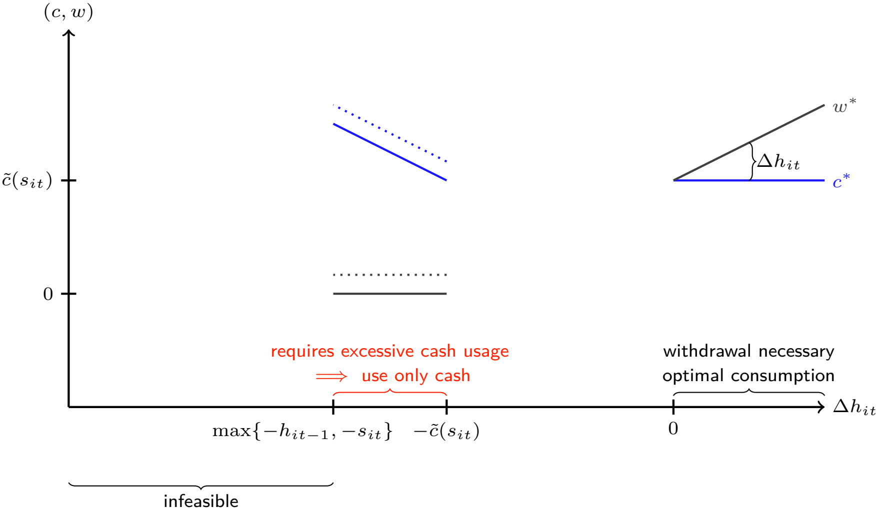


Model: In-period solution:

◂ Return



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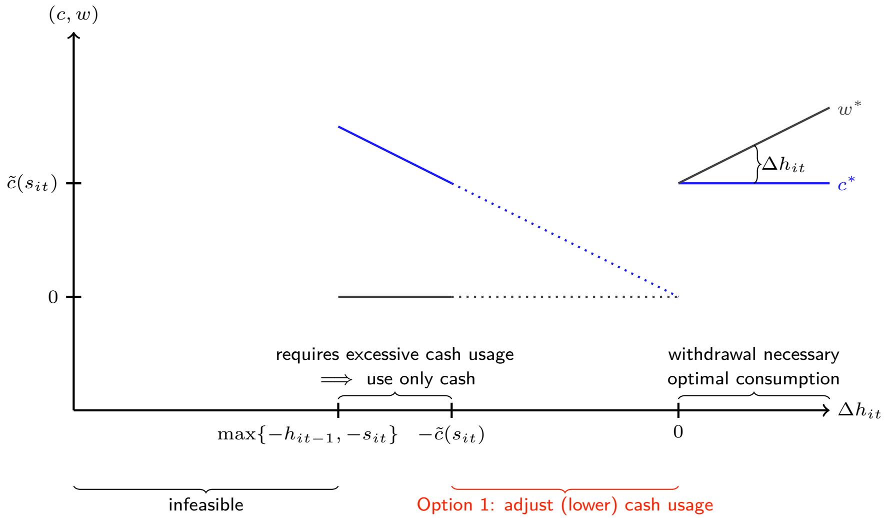

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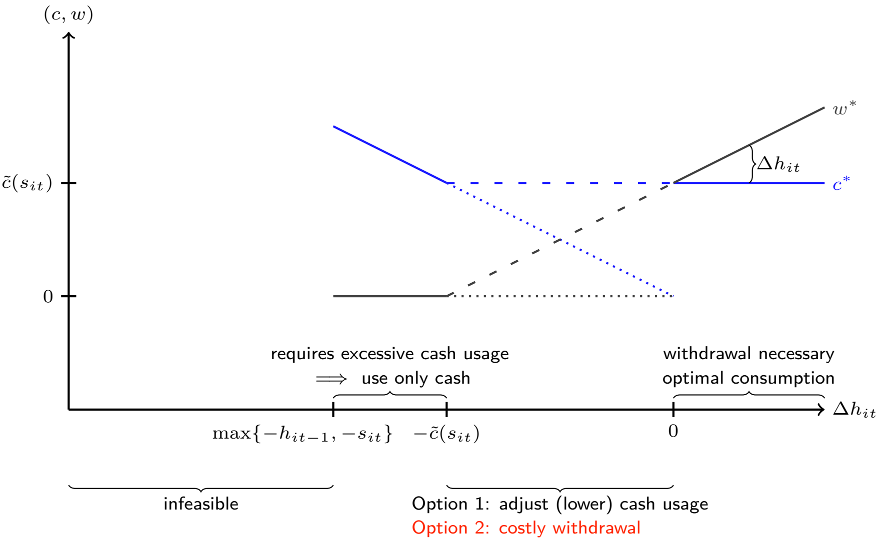

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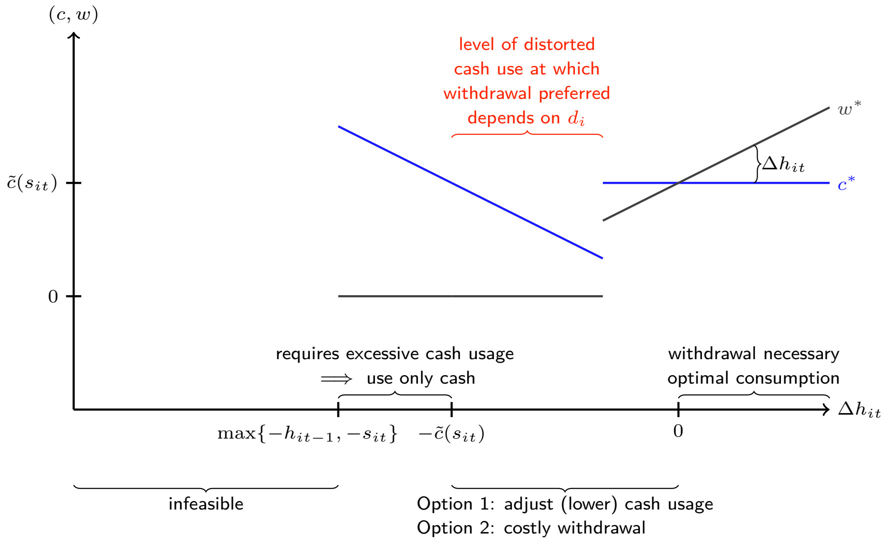
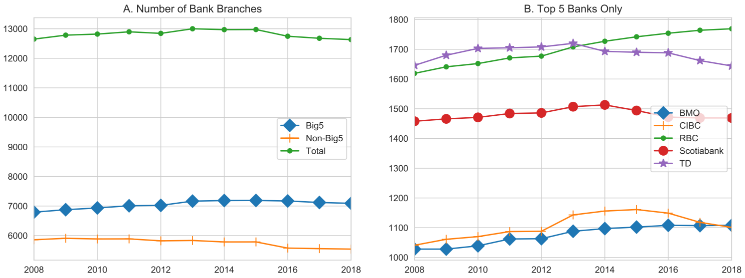


Data: Distance to Bank Network

Return

- d_i reflects density of bank branches in forward sortation area (FSA, 1643 in CAN)
  - $\rightarrow$ $d_i = \frac{\ln(\text{geographic area of FSA})}{\text{\# of banch branches}}$
  - $\rightarrow$ ATMs by FIs typically co-located with branches (88%)
  - $\rightarrow$ white-label ATMs only account for $\approx$ 20% of ATM withdrawals (Chen et al. 2021)



Evolution of bank branches.

Evolution over time by Demographic Group

Return

Table 1: Evolution of parameter estimates by demographic type (median estimates)

	Young & Poor	Young & Rich	Old & Poor	Old & Rich	Urban	Rural
(a) Cash elasticity, α						
2009	0.282	0.240	0.269	0.311	0.259	0.344
2013	0.282	0.216	0.297	0.232	0.257	0.333
2017	0.245	0.235	0.297	0.234	0.245	0.257
(b) Holding cost, γ						
2009	0.00161	0.00162	0.00062	0.00091	0.00112	0.00153
2013	0.00209	0.00150	0.00072	0.00054	0.00080	0.00255
2017	0.00154	0.00134	0.00077	0.00049	0.00082	0.00145
(c) Withdrawal cost, F						
2009	0.257	0.258	0.141	0.209	0.269	0.084
2013	0.298	0.276	0.240	0.223	0.290	0.105
2017	0.394	0.392	0.269	0.296	0.375	0.100
N-obs.	924	749	1189	562	2902	522

Evolution of Infrastructure (Sample)

◂ Return

Year	Mean	p10	p25	Median	p75	p90	N-obs.
2009	288.14	0.71	1.67	6.35	111.66	573.90	973
2013	160.32	0.52	1.24	3.42	40.72	279.70	1408
2017	205.65	0.48	1.24	3.48	38.72	287.62	1043
All	210.50	0.57	1.38	3.95	55.62	349.48	3424

Notes: The distance measure is the natural logarithm of the geographic area of the forward sortation area (FSA) consumers are located in in square kilometers over the number of available bank branches.

Evaluation of Factual Infrastructure Changes: Cash Holdings

Return

Infrastructure	All	Urban	Rural	Y&P	Y&R	O&P	O&R	Substantial*
2009	-	-	-	-	-	-	-	-
2013	0.18%	0.23%	-0.13%	1.16%	0.16%	-0.18%	-0.06%	0.75%
2017	0.13%	0.23%	-0.35%	0.45%	0.70%	-0.15%	-0.09%	2.69%

Elasticity

Return

Elasticity of predictions w.r.t. increase in distance (25% change)

(a) All consumers	mean	min	p10	p25	p50	p75	p90	max
average withdrawal amount	-0.75	-4.00	-4.00	-0.01	0.01	0.09	0.49	2.32
expected withdrawal frequency	-1.19	-4.00	-4.00	-2.06	-0.34	-0.12	-0.05	0.00
average cash holding	-0.61	-4.00	-4.00	-0.13	-0.02	0.00	0.37	3.59
average cash use	-1.09	-4.00	-4.00	-1.81	-0.19	-0.06	-0.01	0.00
expected payoff per period	-0.14	-3.98	-0.30	-0.08	-0.01	-0.00	-0.00	-0.00
Observations	3161							

(b) Cash users (post increase)	mean	min	p10	p25	p50	p75	p90	max
average withdrawal amount	0.16	-0.28	0.00	0.00	0.01	0.24	0.56	2.32
expected withdrawal frequency	-0.39	-3.58	-0.73	-0.44	-0.23	-0.08	-0.04	0.00
average cash holding	-0.00	-3.48	-0.17	-0.06	-0.02	0.00	0.46	3.59
average cash use	-0.27	-3.48	-0.48	-0.27	-0.12	-0.04	-0.01	0.00
expected payoff per period	-0.05	-2.65	-0.19	-0.03	-0.01	-0.00	-0.00	-0.00
Observations	2465							

(c) Cash non-users (post increase)								
expected payoff per period	-0.46	-3.98	-1.46	-0.37	-0.07	-0.00	-0.00	-0.00
Observations	696							

- focus on 25% change
  - → reaffirm bi-modality
  - → 22% no longer use cash
- Who does what?